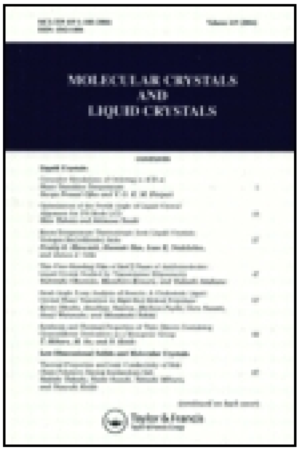

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ACOUSTICAL AND OPTICAL BEAM DISPLACEMENT IN LIQUID CRYSTALS

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The occurence of acoustical and optical beam displacement effects at an interface of a liquid crystal and another medium is explored. In contrast with earlier discussions of infinite plane wave interactions, it is argued that finite beams will undergo observable beam displacements. The amount of participation by differrent regions of liquid crystalline surfaces, contributing to displacement, is evaluated for undisturbed surfaces as well as for those subjected to external fields. Beam displacement effects are shown to have measurable values which are of importance in optical computing using LC components, due to their potential for error generation.

The existence and significance of evanescent waves in liquid crystals has recently been discussed by Moritz.[1,2] These discussions have been limited to consideration of plane wave incidence on infinite half-spaces. While these situations are of general interst, a more practical and relevant problem is that of finite beaminteractions. The question of general finite beam interactions is of substantial importance since optical signal and image processing are evolving into a mature stage, with a recognizable role for liquid crystal devices,[3,4] in particular, devices using the variable grating modes as discussed by Soffer et al.[5] This communication addresses the subject of finite beam effects, and in particular, beam displacement effects in liquid crystals for both electromagnetic and ultrasonic beams.

Acoustical beam displacements occuring in the reflection of ultrasonic beams, with a Gaussian amplitude distribution, has been discussed for liquid-solid interfaces by Breazeale, Adler, and Flax[6] (BAF). BAF demonstrated the beam displacement through Schlieren photography of the ultrasonic beams reflected at water-aluminum and water-brass interfaces. The ultrasonic and optical beam displacement effects were also discussed by Tamir and Bertoni.[7,8] A survey of relevant reviews of ultrasonics[9-11] and light interactions[12-14] with liquid crystals, indicates that the beam displacement question has not been addressed in the context of liquid crystals.

Optical lateral- beam displacement effects at interfaces can be summarized as follows: when finite beam incidence occurs near the critical angle of total reflection, the reflected beam undergoes a substantial lateral shift (Δ) from the position expected in the geometric optics approximation. The analogous situation occurs with ultrasonic interactions at boundaries. The conclusion of TB and BAF indicates that the concept of evanescent waves must be generalized. If we consider two infinite half spaces with an interface at $z=0$, evanescent waves[1] are essentially waves that propagate at the interface and decay exponentially away from it. Considering a homogeneous wave equation $(\nabla^2 + k^2)\psi=0$, for $z<0$, with ψ representing the field components, k the plane waves in the $z<0$ half space, $\psi = \exp(i(k_x x - k_z z - \omega t))$, the dispersion relation $k_z=(k^2-k_x^2)^{\frac{1}{2}}$ holds; for $|k_x|>k$, ψ represents evanescent waves.

These evanescent waves occur, in general, only for discrete real values of k_x which will be labeled k_p. The generalization is in recognizing that the propagation constants k_x may assume a discrete set of complex values $k_p=\beta+i\alpha$. The waves are then termed complex guided waves or leaky waves. They may propagate with decay along the x <u>and</u> z axes, in a lossless medium. While this situation is <u>not</u> admissible for a plane wave with real k, it is quite admissible for an evanescent wave of finite extent to excite a leaky wave field. The imaginary component of k_p, the attenuation coefficient α, thus is not restricted to absorption losses and may represent losses due to leakage of energy from the guiding interface region. It is this leaky wave which offers the explanation for the lateral beam displacement. Figure 1 depicts the beam displacement Δ for an incident bounded beam of width w, as compared to the anticipated geometric optical reflection. The leaky wave associated with k_p continuously leaks energy at an angle $\theta_p = \sin^{-1}(\beta/k)$ into the lower half space $(z<0)$.

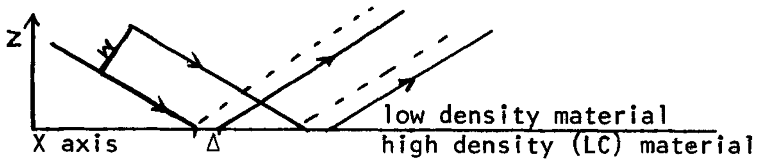


Figure 1: Lateral displacement of reflected beams; solid lines-actual incident & reflected beams, dashed lines- non-displaced idealization.

Liquid crystalline phases offer a diversity of boundary conditions since they present dielectric anisotropies and multilayered structures. As a result of this diversity a variety of forward and backward leaky waves may exist. In this note, emphasis is placed on the forward leaky waves.

The interaction between an incident beam and a leaky wave structure obtains its largest magnitude when the phase matching condition $\sin\theta = \beta/k = \sin\theta_p$ is satisfied, where θ is the angle of incidence. Note that this first-order analysis assumes that interface between the liquid crystal and the second medium is a plane interface with the liquid crystal directors uniformly aligned. The case of fluctuating or distributed director angles is treated in the manner of reference 2.

Following TB, the lateral shifts are dependent on the product of illuminated interface half-width and the attenuation. A useful bounded beam approximation is the Gaussian beam (e.g. laser beam) with an electric field distribution $E(x) = (\pi^{\frac{1}{2}} w)^{-1} \cdot \exp(-x^2/w^2)$ where w is the half beamwidth at the point where $|E|$ decreases to 1/e of its maximum value. The harmonic time dependence $\exp(-i\omega t)$ is implied. The condition of boundedness of the beam will be manifest if the wavelength in the non-liquid crystalline space satisfies the restriction $k_w = 2\pi w/\lambda >> 1$.

The aperture illumination may be represented in the range $-h<z<0$ by a Fourier spectrum composition (for $z=\ell$ and $\alpha=(k^2-\xi^2)^{\frac{1}{2}}$)

$$E_{inc}(x) = \frac{1}{2\pi}\iint F(\eta)\, e^{i\{(\xi_0-\xi) + \eta x + \alpha\ell\}}\, d\xi\, d\eta$$

with F( ) the spectral amplitude which contains the Gaussian distribution. The reflected field may then be represented as

$$E_{ref} = \frac{1}{2\pi}\iint \rho(\xi) F(\eta)\, e^{i\{\phi(\xi) + (\xi_0-\xi)\eta + \xi x + \alpha\ell\}}\, d\eta\, d\xi$$

where $\rho(\xi)$ is the modulus and $\phi(\xi)$ is the phase of the wave reflectance coefficient. At this point, one may desire to discuss multiply layered media such as smectics or periodic cases as obtained through variable grating modes and William's domains.

The salient feature of detailed analysis[6,8] is that the lateral shift of the reflected beam increases with increasing beam widths and attenuation; for a multiply layered structure, the product $\alpha\Delta$ tends to reach a constant value. Δ also increases with a decreasing value of the incidence angle θ. The phase matching conditions, however, must always be satisfied.

Periodic structures (in the xy plane) can be characterized by multiple propagation factors, in contrast with the single value k_p discussed earlier. The multiplicity for integer orders n is expressed through the condition $k_{pn} = k_p + 2\pi n/d$ with d being the periodicity length.

Thus $k_{pn} = \beta_n + i\alpha = \beta + i\alpha + 2\pi n/d$, where β is the phase constant of the fundumental harmonic. The hierarchy of phase matching conditions is $k\sin\theta = k\sin\theta_{pn}$ with θ_{pn} the leakage angle of n-th harmonic. Thus, quite a large number of different lateral shifts are possible since any one of the harmonics may compete, or offer phase matching conditions.

In the optical case, there are possible ordinary and extraordinary wave couplings for leaky fields, as described earlier[2], due to the crystalline aspect (uniaxial, in general, for nematics, biaxial for smectics and biaxial nematics such as those discussed by Saupe[15] and Pleiner and Brand[16]). In the ultrasonic case, there are a variety of propagating and non-propagating modes the leaky waves can couple into.

If the effects of fluctuations in director orientation are now taken into account, in the manner of reference 2, it is obvious that there always will be a certain fraction of liquid-crystalline material whose directors will satisfy the phase matching conditions for excitation of leaky waves and consequent beam displacement.

In order to evaluate the effect of fluctuations, one needs to consider a range of values for the phase matching conditions, i.e., $\theta_{pn} - \delta\theta \leq \theta \leq \theta_{pn} + \delta\theta$, where $\delta\theta$ is an angular acceptance measure for phase matching. For the case of an undisturbed sample, the fraction of a bounded beam that is shifted depends on the distribution of director orientations, and the order (n) that we are interested in (as well as grating periodicities). Adopting the classical Saupe-Maier theory to illustrate the general features of interaction with nematics, the orientational distribution function (see notation of reference 2) is

$$\rho(\cos\alpha)=Z^{-1}\exp\{\frac{A_oS}{2kT}(3\cos^2\alpha-1)\}$$

with

$$Z=\int_0^1 d(\cos\alpha)\exp\{\frac{A_oS}{2kT}(3\cos^2\alpha-1)\}$$

The fraction of shifted energy to non shifted energy is thus propertional to the ratio, Q, of director angles satisfying the phase matching conditions, to all angles. In other words:

$$Q_{pn}(\theta_o,\delta\theta) = Z^{-1}\int_{\cos(\theta_{pn}-\delta\theta-\theta_o)}^{\cos(\theta_{pn}+\delta\theta-\theta_o)}(\cos\alpha)d(\cos\alpha)$$

where the mean orientation of the director is θ_o and the fluctuation angle is α ($\theta=\theta_o+\alpha$). The quantity $\delta\theta$ is the angular acceptance of phase mismatch which will allow the beam displacement to occur. TB point out that an angular acceptance of 10 seconds of arc does not cause any difference in the beam displacement Δ for beam width of 2 mm. The actual values of $\delta\theta$ for liquid crystals remain to be determined. Results of representative calculations for nematics with a Saupe-Maier potential and $A_oS/2kT=2$ are given in figure 2.1 for $\delta\theta$ values of 10, 20 and 30 seconds of arc (cases a,b and c).

These curves yield the percentage of beam displacement of peak value that will occur for a given order of diffraction grating mode, versus different director angle orientations. Note that the discussion revolves about participation amounts rather than the actual values of the reflected fields. The case of n=0 gives a qualitative description of the beam displacement effects in an undisturbed liquid crystal. When, say a nematic, is subjected to external fields (e.g. electric), the director orientation will vary according to geometry and field dependent conditions. As a representative case, the Williams Domain Modes (WDM) are investigated with director orientation dependence as given by Penz and Ford[17]. The director orientation follow the modulation $\theta=\theta_o\sin q_x x\sin q_z z$ where q_z and q_x are wave vector components satisfying certain geometrical conditions imposed by the cell.

The field induced distortions modify the total number of director angles satisfying the beam displacement conditions. The variation of the director due to the external fields is accounted for by integrating over the angle θ_o. In other words, θ_o becomes a continuous variable according to the preceding expression , and $Q_{pn} \rightarrow Q_{INTEG,pn} = \int Q_{pn}(\theta,\delta\theta)d\theta$.

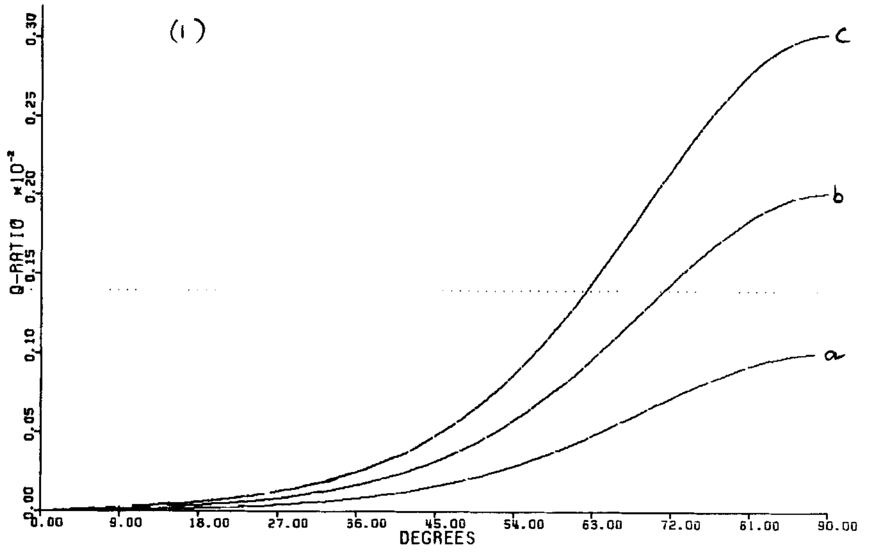
Since we are interested only in surface effects, the z variation does not enter, and we are concerned only with variations along the x axis (e.g. in a display cell where the cell faces are in the xy plane).

When we consider the ratio of Q over a natural length scale and take into account the equivalence of director orientation ($\vec{n}$ and $-\vec{n}$), we see that the integral can be simply expressed as

$$Q_{INTEG,pn}(\theta_o,\delta\theta) = \int_{-\theta_0}^{\theta_0} \left\{ \frac{1}{2} \int_{\cos(\theta_{pn}-\delta\theta-\theta)}^{\cos(\theta_{pn}+\delta\theta-\theta)} \rho(\cos\alpha)\, d(\cos\alpha) \right\} d\theta .$$

To illustrate the effect of the field on theratio of directions participating in the beam shifts, to non participating directions, the grating order dependence θ_{pn} is provisionally suppressed by setting $\theta_{pn}=0$. Figure 2.2 depicts the effect of regular distortion (William's domains) for $\theta_{pn}=0$ and n=0 conditiond corresponding to 10,20 and 30 seconds of arc. While at first it appears that there will be less beam displacement by the field induced disturted surface, it must be recalled that it represents a very specialized situation. The overall beam displacement effects require summation over all grating modes, i.e. $Q_{INTEG,p} = \Sigma_n Q_{INTEG,pn}(\theta_o,\delta\theta)$. The summation is from n=0 to a cutoff mode number determined by the actual boundaries of the grating pattern. It can be seen, however, that for the undistorted situatuion, an asymptotic saturation effect is present, while for the n=0 mode of the field distorted case there is an exponential type increase in the beam displacement participation.

The situation in the acoustic case is expected to be similar. It is apparent, however, that with increasing use of analog computing devices utilizing liquid crystal components, the presence of beam displacement effects, even at a conservative 10^{-5} percent level (suggested by figure 2.2), will introduce computing errors. The actual error rates are not the same as the beam displacement q ratios discussed since actual errors depend on the angles and resolutions required. However, in devices that aim towards very high data rates or with multiple liquid crystalline components, this source of computing errors needs to be recognized.



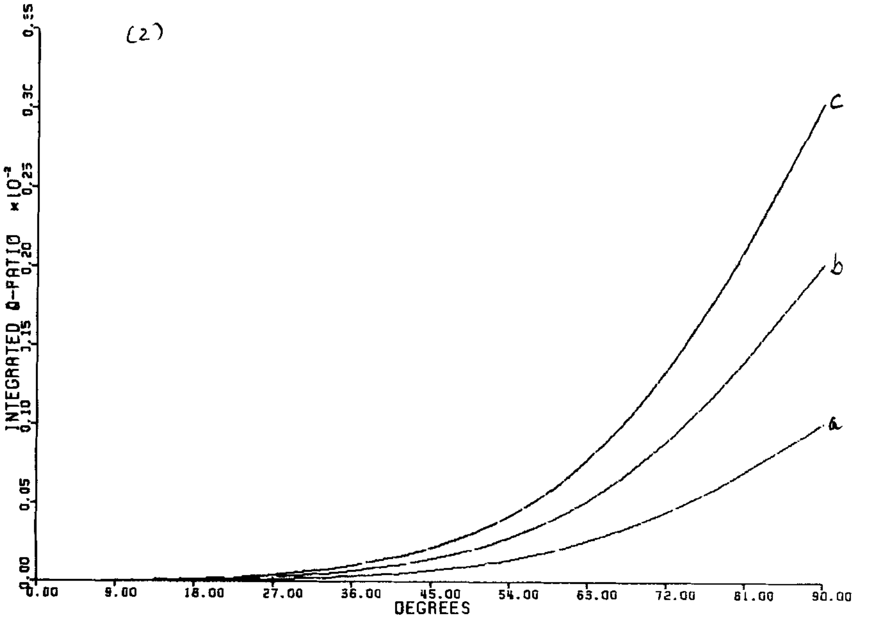


FIGURE 2-(1) Q-ratios, (2) integrated Q ratios; cases a,b,c correspond to $\delta\theta$=10,20, 30 seconds of arc.

REFERENCES

1. E. Moritz, Mol. Cryst. Liq. Cryst. 41(Lett), 63 (1977).
2. E. Moritz, Mol. Cryst. Liq. Cryst. 56(Lett), 177(1980).
3. J.A. Neff, Optical Engineering, 19, 205 (1980).
4. A. Armand, A.A. Sawcchuk, T.C. Strand, D. Boswell and B.H. Soffer, Optics Letters, 5, 129 (1980).
5. B.H. Soffer, D. Boswell, A.M. Lackner and A.R. Tanguay, SPIE Proceedings 232, 128 (1980).
6. M. Breazeale, L. Adler & L. Flax, J. Acoust. Soc. Am., 56, 866 (1974).
7. T. Tamir and H. Bertoni, J Opt. Soc. Am. 61, 1397 (1971).
8. H. Bertoni and T. Tamir, Appl. Phys. 2, 157 (1973).
9. S. Candau and S.V. Letcher, in Advances in Liquid Crystals G.H. Brown,ed., 3, 168 (1980).
10. G.G. Natale, J. Acoust Soc. Am. 63, 1265(1978); ibid p.1677
11. K. Miyano and J.B. Ketterson, in Physica Acoustics, XIV, W.P. Mason, ed.,(Academic Press N.Y.1979).
12. G.H. Brown, J.W. Doane and V.D. Neff, Crit. Rev. Solid State Sci., 1, 303 (1970).
13. P.G. DeGennes, The Physics of Liquid Crystals, (Clarendon Press, Oxford, 1975).
14. H. Kelker and R. Hatz, Handbook of Liquid Crystals, (Verlag- Chemie, Weinham 1980).
15. A. Saupe, J. Chem. Phys., 75, 5118 (1981).
16. H. Brand and H. Pleiner, Phys. Rev. A24, 2777 (1981).
17. P.A. Penz and G.W. Ford. Phys. Rev. A6, 414 (1972).